

Prabhat Academy

HALF YEARLY EXAM - 2019 - 20

CLASS - XI

MATHEMATICS

[M.M.:100]

TIME

Note: Attempt all questions.

1. (a) If an arc of length s subtends an angle θ radians at the centre of a circle of radius r , then $s = r\theta$
- (b) In a circle of diameter 40cm, the length of the chord is 20cm. Find the length of the minor arc of the circle.
2. (a) Find the angle in radians between the hands of the clock at 3:30 A.M.
- (b) If $\cos x + \sin x = \sqrt{2} \cos x$
Show that $\cos x - \sin x = \sqrt{2} \sin x$
Prove that
3. (a)
$$\frac{\cot x + \operatorname{cosec} x - 1}{\cot x - \operatorname{cosec} x + 1} = \frac{1 + \cos x}{\sin x}$$
- (b) If $\sin \alpha$ and $\cos \alpha$ are roots of the equation $ax^2 - bx + c = 0$, then find the relation on satisfied by a , b and c .
4. (a) Find the range of the following functions:
 - (i) $f(x) = 2 - 3\cos x$
 - (ii) $f(x) = 2 + 5\sin 3x$
- (b) If x , y show that the equation $2xy\sin^2\theta = x^2 + y^2$ is possible only when $x = y \neq 0$
5. (a) Draw the graphs of the following functions:
 - (i) $\sin 3x$
 - (ii) $3\sin x$
- (b) Which is bigger? $\sin 55^\circ$ or $\cos 55^\circ$
6. (a) If $\cos A = \frac{1}{7}$ and $\cos B = \frac{13}{14}$, and A , B lie in the first quadrant,
prove that $A - B = 60^\circ$
- (b) If $\cos(x + 2y) = m \cos x$
Prove that $\cot y = \frac{1+m}{1-m} \tan(x + y)$
7. (a) In a triangle ABC , if $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$,
then prove that $\frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$

P.T.O.

(2)

(b) If $\sin A = x \sin(2B - A)$

show that
$$\tan A = \frac{2x \tan B}{(x+1) - (x-1)\tan^2 B}$$

8. (a) Using the principles of Mathematical induction, prove that:

$$1.2 + 2.2^2 + 3.2^3 + \dots + n.2^n = (n-1)2^{n+1} + 2$$

(b) If z_1 and z_2 are two complex numbers such that $\frac{2z_1}{3z_2}$ is purely imaginary numbers

then find the value of $\left| \frac{z_1 - z_2}{z_1 + z_2} \right|$

9. (a) Convert the following complex numbers in the polar form and represent them in Arg and Plane.

(i) i

(ii) $-3i$

(b) Convert in polar form:

(i) $\frac{(\sqrt{3}-1) - (\sqrt{3}+1)i}{2\sqrt{2}}$

(ii) $i(1+i)$

10. (a) Which term of the sequence $25, 24^{1/4}, 23^{1/2}, 22^{3/4}, \dots$ is the first negative term.

(b) Which term of the sequence $12+8i, 10+7i, 8+6i, \dots$ is

(i) Real

(ii) Purely imaginary